Bayesian Reconstruction in Lévy Distribution

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Abstract

Let \( Y_1, \cdots, Y_n \) be order statistics from a Lévy distribution with parameter \( \theta \). Assume that some of middle order statistics are lost, that is we only observed the data set \( Y = \{Y_1, \cdots, Y_r, Y_s, \cdots, Y_n\} \). We are going to reconstruct the \( (r < l < s) \)th order statistic based on the data set \( Y \). It should be noted that variance of order statistics of Lévy distribution do not exist. Therefore, we limit ourselves to use the order statistics for which the moments exist. The number of these order statistics can be seen Theorem 2.1 of [1]. In this work, we propose two methods: Maximum Likelihood Reconstruction(MLR) and Bayesian Reconstruction(BR). To compute MLR, if \( \theta \) be known, we obtain closed form for reconstruction \( Y_l \). However, if \( \theta \) is unknown, in this case, first we obtain MLE’s \( \theta \) by EM algorithm, then reconstructed \( Y_l \) by maximum likelihood approach. The next method for reconstruction \( Y_l \) is the Bayesian approach. In this approach, we assume that the unknown parameter \( \theta \) is viewed as the realization of a random variable which has a prior distribution. Let us denote the posterior distribution of \( \theta \) by \( \pi(\theta|y) \). Then, the Bayes reconstructor of \( Y_l \) under the squared error loss function is as follows

\[
BR = \int_{\theta_y} \int_{y} y_l f(y_l|y, \theta)\pi(\theta|y)d\theta dy. \tag{1}
\]

A numerical method has to be applied to compute the BR. Also, numerical example and Monte Carlo simulation study of the Lévy distribution are given to illustrate all the reconstruction methods discussed in this work. To evaluate the estimators, we compute Mean Square Reconstruction Errors for the MLR and BR reconstructions. Finally, we conclude BR is better than MLR.

Keywords: Bayes estimator; EM algorithm; reconstruction point; order statistics; Lévy distribution.

References


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