Robust Burg Estimation of stationary autoregressive mixtures covariance

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1 Introduction

In many engineering applications, non-Gaussian models are used for example for the modelization of I&Q data of strong clutters in radar such as ground or sea clutters. The family of complex elliptically symmetric distributions (which contains a lot of classical distributions such as multivariate Gaussian, multivariate Cauchy distributions and multivariate K-distributions) is a useful generalization of Gaussian random vectors by keeping the shape and location parameters.

For our applications, the location parameter will be zero, so if we suppose that the clutter is homogeneous, the key point is the estimation of the scatter matrix of the clutter through samples \(x_1, \ldots, x_N \in \mathbb{C}^d\). The family of elliptical distributions is well adapted for the modelization non-Gaussian samples [7] by its generality given by its non-parametric amplitude part. In this framework, we will aim two kind of robustness for the estimation of the scatter matrix:

- a robustness with respect to the amplitude distribution which is often heavy-tailed
- a robustness in case of inhomogeneous distribution i.e. the samples are contaminated

For general elliptical distributions, many estimators were proposed and among them, a M-estimator [9] proposed by Tyler which was shown to be a maximum likelihood estimator for normalized samples \(\frac{x_1}{\|x_1\|}, \ldots, \frac{x_N}{\|x_N\|}\). Unfortunately, these estimators often lack of the second robustness listed above. We propose then to inspire from robust Huber method for the estimation of the scatter matrix of the samples \(\frac{x_1}{\|x_1\|}, \ldots, \frac{x_N}{\|x_N\|}\) that were shown to share a distribution called angular central Gaussian [10].

Furthermore, we will consider stationary signals which adds a Toeplitz structure constraint for the scatter matrix. For Gaussian autoregressive vectors, it is usual to use Burg methods in order to estimate Toeplitz covariance matrices. We propose to adapt these techniques for mixtures of autoregressive vectors, a big subfamily of the class of elliptical distribution. Estimators that take into account the Toeplitz constraints in the context of general elliptical distributions were proposed by [8].

We will compare our estimators to a geometrical method consisting in computing the median of Gaussian autoregressive models estimated from each sample \(x_i\) [1] [2][3]. This method was shown to resist to the second robustness aimed thanks to the induced Riemannian geometry of the parameters.

In the first part, we will explicit the model we consider for the samples. Then, we will present the estimators on normalized samples in the general case. In a third part, we will adapt classical Burg methods for autoregressive mixtures in order to estimate the constrained scatter matrix and we will conclude by presenting some illustrations on simulations.
References


