We describe a generalization of the TIB representation of a time invariant linear system from scalar (SISO) to vector (MIMO) case. Several advantageous features of the scalar case, such as numerical stability, fast state update and efficient dimensionality reduction, are also extended to the matrix case. Consider a $d$-dimensional innovations model,

$$z(t+1) = Az(t) + Bx(t),$$

$$y(t) = Cz(t) + x(t),$$

with $y(t)$ being a sequence of $d$-dimensional measurement vectors and $z(t)$ being $n$-dimensional state vectors. By appropriate choice of state-space coordinates we choose $A$ to be a lower triangular, and input balanced: $AA^* + BB^* = I$. As a special case of orthogonal filter, input balance ensures good numerical performance, and the triangular realization has high precision location of eigenvalues and a band matrix fraction representation which affords fast algorithms and a very efficient model reduction algorithm for both SISO and MIMO cases is also known for the TIB parametrization.

According to Douglas-Shapiro-Shields factorization, any strictly proper rational stable function can be factored into an unstable part and rational lossless part, where for the TIB realization the unstable part is related to $C$ and the lossless part is associated with $A$ and $B$. Once the lossless part has been determined, the transfer functions have convenient geometry. The Schur algorithm constructs scalar rational lossless functions, the extension to the MIMO case is the Schur tangential algorithm,

$$\mathcal{B}_i(1/\bar{w}_i) u_i = v_i,$$

where $\mathcal{B}_i$ is a sequence of rational lossless functions with degree/McMillan degree $i$. When $v_i = 0$, then $w_i$ are the poles of the system, and $\mathcal{B}_i$ are the Blaschke-Potapov factorization in MIMO case (and Blaschke factors in the SISO case). The TIB pair $(A, B)$ is uniquely determined by the poles of the system for SISO, whereas for MIMO, the null vectors $u_i$ are also required to determine the lossless part. The reduction algorithm we propose is an hybrid one, which obtains the lossless part by minimizing the Hankel ($H^\infty$) norm and the unstable part by minimizing $H^2$ norm. The lossless part $(A, B)$ can be determined from a partial...
SVD of the Hankel matrix given by

\[ H_{ij} = f_{i+j-1}, i, j = 0, 1, \ldots \]  

(4)

where \( \{ f_i \} \) are the impulse responses. \( C \) can be therefore obtained by a well conditioned least squares regression.

In the SISO case, the choice of model parameters as the power series coefficients of the logarithm of the transfer function results in the Fisher information matrix is an identity matrix, providing a Euclidean statistical (Hilbert) manifold. Denoting \( \log f(z) = a_0 + a_1 z + a_2 z^2 + \cdots \), since \( f \) and \( \log f \) has the same singularity, we can apply our reduction algorithm on the alternative Hankel matrix

\[ A_{ij} = a_{i+j-1}, i, j = 0, 1, \ldots \]  

(5)

and then recover the parameters after exponentiation. Mullhaupt and Choi proved in the SISO case, the information geometry is determined by the unstable part of the transfer function, which our algorithm can extend to the MIMO case.

References