Endometriosis: MRI navigation and surface reconstruction on manifolds

GSI2015

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What is endometriosis?

10% of women

Ovaries • Uterosacral ligaments • Colon • vagina • bladder
How to diagnose and cure?

Before surgery
location • size • depth

Question 1: How to merge both techniques?
Question 2: How to evaluate the size of the cyst?
When endometriosis meets manifolds

**Answer 1:**
MRI navigation as a path on SE(3)
When endometriosis meets manifolds

Answer 2:
Endometrial volume reconstruction as path on shape manifold
How to interpolate points on manifolds?
How to interpolate?

Each segment between two consecutive points is a Bézier function.
Reconstruction: the De Casteljau algorithm

\[ \beta_2(b_0, b_1, b_2; 34) \]

\[ \beta_2(b_0, b_1, b_2; 12) \]

\[ \beta_2(b_0, b_1, b_2; 04) \]
Reconstruction: the De Casteljau algorithm

\[ \beta_2(b_0, b_1, b_2; 14) \]

\[ \beta_2(b_0, b_1, b_2; 12) \]

\[ \beta_2(b_0, b_1, b_2; 34) \]
Reconstruction: the De Casteljau algorithm

\[ \beta_2(b_0, b_1, b_2; t) \]

\[ \beta_2(b_0, b_1, b_2; t) \]

\[ \beta_2(b_0, b_1, b_2; t) \]
Reconstruction: the De Casteljau algorithm
Reconstruction: the De Casteljau algorithm

\[ b_1 = (b_0, b_1, b_2; t) \]

\[ b_2 = (b_0, b_1, b_2; t) \]

\[ b_2 = (b_0, b_1, b_2; t) \]
Reconstruction: the De Casteljau algorithm

\[ \beta_2(b_0, b_1, b_2; 1) \]

\[ \beta_2(b_0, b_1, b_2; 1/2) \]

\[ \beta_2(b_0, b_1, b_2; 3/4) \]
Reconstruction: the De Casteljau algorithm
Reconstruction: the De Casteljau algorithm

\[ b_1 \left( b_0, b_1, b_2; \frac{1}{4} \right) \]

\[ b_1 \left( b_0, b_1, b_2; \frac{1}{2} \right) \]

\[ b_1 \left( b_0, b_1, b_2; \frac{3}{4} \right) \]
Reconstruction: the De Casteljau algorithm

\[ \beta_2(b_0, b_1, b_2; \frac{1}{4}) \]
Reconstruction: the De Casteljau algorithm

\[ \beta_2(b_0, b_1, b_2; \frac{1}{4}) \]

\[ \beta_2(b_0, b_1, b_2; \frac{3}{4}) \]

\[ \beta_2(b_0, b_1, b_2; 1) \]
Reconstruction: the De Casteljau algorithm

\[ \beta_2(b_0, b_1, b_2; \frac{1}{4}) \]
Reconstruction: the De Casteljau algorithm

\[ \beta_2(b_0, b_1, b_2; \frac{1}{4}) \]

Diagram showing the De Casteljau algorithm for a quadratic Bezier curve with control points \( b_0, b_1, b_2 \). The algorithm iterates between points along the curve, with the control points marked at \( t = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1 \).
Reconstruction: the De Casteljau algorithm

\[ \beta_2(b_0, b_1, b_2; \frac{1}{4}) \]
Reconstruction: the De Casteljau algorithm

\[ \beta_2(b_0, b_1, b_2; \frac{1}{4}) \]

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\[ \beta_2(b_0, b_1, b_2; \frac{3}{4}) \]

\[ \beta_2(b_0, b_1, b_2; \frac{1}{2}) \]
Example on the sphere

It's ugly. Make it smooth!
Example on the sphere

It’s ugly. Make it smooth!
Smooth interpolation with Bézier (in $\mathbb{R}^n$)
Smooth interpolation with Bézier (in $\mathbb{R}^n$)

Find the optimal position of control points
$C^1$-piecewise Bézier interpolation (in $\mathbb{R}^n$)
$C^1$-piecewise Bézier interpolation (in $\mathbb{R}^n$)

\[ p_{i-1} - b_i^+ = 2p_i - b_i^- \]
Optimal $C^1$-piecewise Bézier interpolation (in $\mathbb{R}^n$)

Minimization of the mean square acceleration of the path

$$\min_{\alpha_i} \int_0^1 \|\dddot{\beta}_2^0(b_1^-; t)\|^2 dt + \sum_{i=1}^{n-1} \int_0^1 \|\dddot{\beta}_3^i(b_i^-; t)\|^2 dt + \int_0^1 \|\dddot{\beta}_2^n(b_{n-1}^-; t)\|^2 dt$$
Optimal $C^1$-piecewise Bézier interpolation (in $\mathbb{R}^n$)

Minimization of the mean square acceleration of the path

$$\min_{\alpha_i} \int_0^1 \|\dddot{\beta}_2(b^-_1; t)\|^2 dt + \sum_{i=1}^{n-1} \int_0^1 \|\dddot{\beta}_3(b^-_i; t)\|^2 dt + \int_0^1 \|\dddot{\beta}_2(b^-_{n-1}; t)\|^2 dt$$

Second order polynomial $P(b^-_i)$
Optimal \( C^1 \)-piecewise Bézier interpolation (in \( \mathbb{R}^n \))

Minimization of the mean square acceleration of the path

\[
\min_{\alpha_i} \int_0^1 \| \dddot{\beta}^0_i(b_1^-; t) \|^2 dt + \sum_{i=1}^{n-1} \int_0^1 \| \dddot{\beta}^i_i(b_i^-; t) \|^2 dt + \int_0^1 \| \dddot{\beta}^n_i(b_{n-1}^-; t) \|^2 dt
\]

Second order polynomial \( P(b_i^-) \)

\[
\nabla P(b_i^-)!
\]
Optimal $C^1$-piecewise Bézier interpolation (in $\mathbb{R}^n$)

Minimization of the mean square acceleration of the path

$$\min_{\alpha_i} \int_0^1 \|\ddot{\beta}_0^0(b_1^--t)\|^2 dt + \sum_{i=1}^{n-1} \int_0^1 \|\ddot{\beta}_3^i(b_i^--t)\|^2 dt + \int_0^1 \|\ddot{\beta}_2^n(b_{n-1}^--t)\|^2 dt$$

Second order polynomial $P(b_i^-)$
Optimal $C^1$-piecewise Bézier interpolation (in $\mathbb{R}^n$)

Minimization of the mean square acceleration of the path

\[ B^{-} = A^{-1} CP = DP \quad \Leftrightarrow \quad b_i^{-} = \sum_{j=0}^{n} D_{ij} p_j \]
A result on $\mathbb{R}^2$
Optimal $C^1$-piecewise Bézier interpolation (on $\mathcal{M}$)

- The control points are given by:

$$b_i^- = \sum_{j=0}^{n} D_{i,j} p_j$$
Optimal $C^1$-piecewise Bézier interpolation (on $\mathcal{M}$)

- The control points are given by:

$$b_i^- = \sum_{j=0}^{n} D_{i,j} p_j$$

- These points are invariant under translation, *i.e.*:

$$b_i^- - p^{\text{ref}} = \sum_{j=0}^{n} D_{i,j} (p_j - p^{\text{ref}})$$
Optimal $C^1$-piecewise Bézier interpolation (on $\mathcal{M}$)

- The control points are given by:

$$b_i^- = \sum_{j=0}^{n} D_{i,j} p_j$$

- These points are invariant under translation, i.e.:

$$b_i^- - p^{ref} = \sum_{j=0}^{n} D_{i,j} (p_j - p^{ref})$$

- Transfer to the manifolds setting using the Log as $a - b \Leftrightarrow \text{Log}_b(a)$

$$\text{Log}_{p^{ref}}(b_i^-) = \sum_{j=0}^{n} D_{i,j} \text{Log}_{p^{ref}}(p_j)$$
Application 1: MRI navigation

Monitor

Ultrasound transducer

Bézier path

\[
\begin{align*}
R_3 \circ SO(3) \\
S_2 \circ SO(3)
\end{align*}
\]
Application 2: Endometrial volume reconstruction
Conclusions

General $C^1$-interpolative method on manifolds... applied in medical imaging.
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General $C^1$-interpolative method on manifolds... applied in medical imaging.

- It’s light;
- It’s fast;
- It’s general;
Conclusions

General $C^1$-interpolative method on manifolds... applied in medical imaging.

- It’s light;
- It’s fast;
- It’s general;
- Bézier interpolation can be extended to multidimensional interpolation (surfaces);
Any questions?

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