Similitude and non symmetric geometry for dispersion modelling

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Abstract—A low complexity model for the transport of passive scalars is presented. It can be applied for environmental analysis, especially regarding the atmospheric drift of agricultural pesticides. A multi-level approach has been carried out with a reduced dimension of the solution space at each level. Similitude solutions are used in a non symmetric metric for the transport over long distances. The calculation does not require any PDE and is mesh free. The model also permits to access the solution in one point without computing the solution over the whole domain.

Index Terms—Non symmetric geometry, Reduced order modelling, Risk analysis by sensitivity, Source identification Topography optimisation.

Résumé—Un modèle à complexité réduite dédié au transport de scalaires passif est présenté. Il peut être appliqué aux analyses environnementales, notamment par rapport à la dérive atmosphérique des pesticides d’origine agricole. Une approche multi-niveaux a été menée avec une réduction de l’espace de solution à chaque niveau. Des solutions de similitudes sont employées selon une métrique non symétrique pour le transport sur de longues distances. Les calculs ne nécessitent pas la résolution d’équations aux dérivées partielles et fonctionnent sans maillage, comme pour les simulations CFD traditionnelles. Le modèle permet aussi d’accéder à la solution en un point sans avoir à calculer la solution sur l’ensemble du domaine. Cet article présente le modèle de dispersion utilisé et aborde les perspectives envisageables pour ce travail, notamment, vis-à-vis de son couplage avec les systèmes d’information géographique (SIG) et les technologies géomatiques.

Mots clés—Géométrie non symétrique, Modélisation à complexité réduite, analyse de risques, Identification de source de pollution, Optimisation de la topographie, Couplage SIG.

I. INTRODUCTION

Air pollution due to pesticides is a major concern today for human health and ecosystems. One aims to model pesticide transport in atmospheric flows with very low calculation cost using assimilation-simulation and statistic analysis. As the meteorological data is incomplete and is difficult to acquire at field, but also presents a high variability and a huge variety of parameters, the reduced order modelling appears as a natural way to proceed in our case.

Our goal is to build a multi-level model where a given level provides the inlet condition for the level above. A near field (to the injection device) search space has first been built [9] using an adaptation of the jet flow theory and field experimental observations [1] [2]. The results of this local solution give the amount of specie leaving the atmospheric sub-layer, over the plant rows [3]. This quantity is candidate for the long range transport over plots. The long distance transport is the last step of the model and its calculation is detailed in this paper, regarding similitude solutions designing and layers and plumes mixing [20], as known in the Cartesian metrics.

II. REDUCED ORDER MODELLING

One aims to model a large multi-scale phenomenon induced by agricultural phyto-treatments applied on vineyards. As this process induces several stages of uncertainty and presents a huge variability in space and time [9], only the most important drift parameters are studied. The relevant entities taken into account are the density and orientation of the plant rows, the local topography which is given by an accurate digital elevation model and the atmospheric conditions such as wind velocity and direction. Some of the input data are missing and are chosen “a priori” [4] in both literature and field experiments. Despite this lack of data, the model is designed to forecast the concentrations over a five square kilometres domain with quite uniform weather conditions. Future prospects tend to validate the model with some real embedded meteorological data.

A. Long range transport and non symmetric geometry

The results of the near-field level provide a local distribution of the advected quantities of pesticides. We are now interested to model quantities ready for a transport over large distances, equivalent to the agricultural watershed scale. We suppose that those are given by:

\[ c^+(x,y)=\int_{z>H} c\, dz \quad \text{or} \quad c^+(x,y)=u^+\cdot c \]

Where \( H \sim 2-3 \ m \) and \( u^+=\max(0, (u.z)/\|u\|) \)

The total quantity being transported is then given by the following integral:

\[ C = \int_{IR^2} c^+(x, y) \, ds \]
One now aims to reduce the search space for the solution, and assumes that the plume advects downwind and spreads out in the horizontal and vertical directions. Hence, the distribution of a passive scalar $c$, emitted from a given point and transported by a uniform plane flow field $U$ along $x$ coordinate, is given by:

$$c(x, y, z) = c_c(x) f(\sqrt{y^2 + z^2}, \delta(x))$$

Where $c_c(x) = \exp(-a(U)x)$ and $f(\sqrt{y^2 + z^2}, \delta(x)) = \exp(-b(U, \delta(x)) \sqrt{y^2 + z^2})$

$c_c$ describes the behaviour along the central axis of the distribution and $\delta(x)$ characterizes its thickness at a given $x$ coordinate. An analogy exists with plane or axisymmetric mixing layers and neutral plumes where $\delta(x)$ is parabolic for a laminar jet and linear in turbulent cases. $a(.)$ is a positive monotonic decreasing function and $b(,.)$ is a positive monotonic one which increases in $U$ and decreases in $\delta$. In a uniform atmospheric flow field, this solution using the Euclidian distance can be used for the transport of $c^+$ above the crops, as shown on the figure 1 below.

\[C_c(x) \text{: Concentration along the plume axis} \]
\[\delta(x) \text{: Lateral distance. Thickness of the plume.} \]
\[\text{Fig. 1: Scheme of the plume using the Euclidian similitude} \]

### B. Non symmetric geometry

Using the Euclidian metric system, a distance function between two points $A$ and $B$ verifies the following relationship:

$$d(A,B) = 0 \Rightarrow A = B, \quad d(A,B) = d(B,A), \quad d(A,B) < d(A,C) + d(C,B)$$

In our case, we have chosen an original Riemannian metric $M$, according to which the distance between $A$ and $B$ is given by:

$$d_M(AB) = \int_0^T \left(\sqrt{\dot{AB}M + AB}^2\right)^{1/2} dt$$

where $M$ is positive and symmetric. If $M=I$ with $I$ as the identity matrix, one can recover the Euclidean geometry. The variable $M$ allows accounting for anisotropy and non uniformity of the distance function, as shown on figure 2. Let’s consider now the following distance function definition:

If $A$ is upwind with respect to $B$ then:

$$d(B,A) = \infty \quad \text{and} \quad d(A,B) = \int_A^{B_\perp} ds/u = T$$

$T$ defines the migration time from $A$ to $B_\perp$ along the streamline passing by $A$. The local velocity along this characteristic is given by $u$, and the latter is by definition tangent to the streamline. $B_\perp$ denotes the projection of $B$ over this characteristic in the Euclidean system. One supposes that the streamline is unique and avoids sources and attraction points in the flow field. In case of non uniqueness of this projection, one chooses the direction of the projection which satisfies best the constraint $u \cdot \nabla C_g = 0$ in $B$.

Using this new approach based on the time transport, as shown on figure 2, the concentration distribution can be given by:

$$C(x, y) = cc(s)f(n, d(s))$$

\[Cc(S) \text{: Concentration along the plume axis} \]
\[(s,n) \text{: local coordinates on the streamline} \]
\[\text{Fig. 2: Scheme of the Riemannian similitude} \]

### C. Generalized Plume solution

Once this distance built, one assumes that the distribution of a passive scalar transported by a flow $u$ can be written as:

$$c_{\perp} = c_c(S)f(\perp E, \delta(d))$$

The subscript $g$ reads for global and mentions long distance...
transport. $d_E$ is equal to the Euclidean distance in the local normal direction to the streamline at $B \perp (i.e.,$ along direction $B \perp ) \{5, 6, 7, 8\}$. 

D. Calculation of migration times

As we said, our approach aims to provide the solution at a given point without calculating the whole solution. Thus, the model does not require any meshing. Being in point $B$, one needs an estimation of the migration time from the source in $A$ to $B$. We started from a third order polynomial function verifying for each coordinate:

$P_n(0) = x_n, P_n(1) = x_n, P'_n(0) = u'_n, P'_n(1) = u'_n$

If $P'_n(\zeta) \neq u'(x=P_n(\zeta))$, this new point should be assimilated by the construction increasing by one the polynomial order. $\zeta \in ]0,1[$ and is chosen randomly.

Knowing a given constructed flow field (figure 4), the model is able to quickly compute both a concentration plume from a plot (figure 5: middle), and the inverse problem to find the possible origin of a given pollution (figure 5: right).

Fig. 5: Left: constructed wind field. Middle: dispersion of the concentration from a vineyard. Right: sensitivity analysis for a dispersion detected on the lower left corner

III. CONCLUDING REMARKS

A low-complexity model has been presented for the prediction of the transport of a passive scalar dispersion in atmospheric flows in an agricultural context. A non-symmetric metric based on migration times has been used to generalize injection and plume similitude solutions in the context of variable flow fields. Data assimilation has been used to define the flow field and the parameters in the dispersion model. Sensitivity analysis also has been coupled with the low-complexity modelling to introduce robustness issues in the prediction of the atmospheric concentrations of pesticides.

The future needs of our study will concern the optimisation and the validation of the model, especially regarding terrain and meteorological variables. For example, it will be interesting to improve the calculation of the impacts of the topography on the drift, like some of the ground variations ($(x,y,z) \rightarrow \delta(x,y,z)$) in the prediction of the model given above. The "$\delta$" values are available from digital elevation models (DEM) \{13\} produced by GIS programs. The future works will be based on the optimisation of the construction of flow fields according to the local topography, in the aim to obtain a better interaction between the terrain and wind conditions, and to modify the drift calculation. This will be done using the Newton low on the potential flow, and the principles of GIS technologies that we want to couple with the whole system.

Once those optimisations will be done, it will become possible to calculate and visualize the concentrations cloud in its real geographic environment as presented in figure 6. Coupling the drift model with GIS software and algorithms would probably be helpful to apply the calculation on real input datasets and to build realistic time-series for the whole drift process. This way, one would be able to easily build some pollution maps and to proceed to pollution risk analysis.

Fig. 3: Examples of Euclidean (left) and nonsymmetric (right) distances. Same wind field and source term located at the top right of the figures. Lines are isochrone.

Fig. 4: Construction of the wind field at local and global scales

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using the cloud as a geographic object and making it interact with other relevant GIS layers.

**Fig. 6: 3D Visualisation of the concentration cloud as a vectorial shapefile (.shp) overlaying an ortho-image**

**Fig. 7: Details of the resulted GIS Gaussian plume with a continuous colour rendering applied on atmospheric concentrations**

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