Triangulating Statistical Manifolds

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In this paper, we describe an algorithm to construct an intrinsic Delaunay triangulation of a smooth closed statistical manifold, i.e. a manifold of probability density functions (PDFs). In different applications (namely in data analysis, information geometry, biology, shape analysis, compression, scientific visualization), statistical data are often seen as points in a metric space, which is most frequently high dimensional. The generally accepted hypothesis is that, although it is embedded in spaces of high dimensions, data lives usually close to a much smaller structure with a low intrinsic dimension. Making use of this intrinsic geometric structure, this paper will develop a certified algorithm that can reconstruct statistical manifolds with a complexity that depends only linearly on the ambient dimension.

We consider the Fisher information metric defined as a particular Riemannian metric on smooth statistical manifolds to calculate the informational difference between measurements. Interestingly, this metric can be understood as an infinitesimal form of the relative entropy of distributions; more specifically, it is the Hessian of the Kullback-Leibler divergence which is a particular instance of well-studied Bregman divergences for which efficient reconstruction algorithms have been proposed [1].

Our algorithm builds over existing algorithms for Bregman Delaunay triangulations [1] and manifold reconstruction using the tangential Delaunay complex proposed in [2]. The central idea is to define Bregman Delaunay triangulations locally and to glue these local triangulations together by removing inconsistencies between them. We view the inconsistencies as arising from instability in the Bregman Delaunay triangulations, and exploit and adapt the results presented in [3] for the Euclidean case. In particular, our technique heavily uses duality. Our main result is an algorithm whose complexity depends only linearly on the ambient dimension, and produces a Bregman Delaunay complex which is guaranteed to be a triangulation of the manifold under appropriate sampling conditions. We also demonstrate that the resulting triangulation coincides with the intrinsic Delaunay complex for the Fisher metric.

References

